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M. M. Weiner

Validation of the
Numerical Electro-
magnetics Code (NEC)
for Antenna Wire
Elements in Proximity
to Earth

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ABSTRACT

This paper summarizes recent MITRE efforts to validate the NEC-3, NEC-GS, and NEC-3I versions of the Numerical Electromagnetics Code (NEC) developed by Lawrence Livermore National Laboratory for predicting the performance of antenna wire elements in close proximity to flat earth. In an early version (NEC-1), the effect of the air-ground interface was included by applying a plane-wave Fresnel reflection coefficient approximation to the field of a point source. The NEC-2 version, while still retaining the Fresnel reflection coefficient model as an option, provides a more accurate ground model by numerically evaluating Sommerfeld integrals.

The version NEC-3 extends the NEC-2 version to cases for bare wire segments below the air-earth interface. Version NEC-GS utilizes rotational symmetry to provide a more efficient version of NEC-3 for the case of a monopole element with a uniform radial wire ground-screen (GS). Version NEC-3I extends NEC-3 to include the case of insulated (I) wires.

ACKNOWLEDGMENTS

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SECTION 1

INTRODUCTION

The Numerical Electromagnetics Code (NEC) is a method-of-moments computer program developed by Lawrence Livermore National Laboratory (LLNL) for predicting the performance of wire-element antennas above or buried in flat earth [1, 2]. In an early version (NEC-1), the effect of the air-ground interface was included by applying a plane-wave Fresnel reflection coefficient approximation to the field of a point source [3, 4]. The NEC-2 version, while still retaining the Fresnel reflection coefficient model as an option, provides a more accurate ground model by numerically evaluating Sommerfeld integrals [1, 2]. Version NEC-3 extends the NEC-2 version to cases where bare wire segments are below the air-earth interface [5]. Version NEC-GS is a more efficient version of NEC-3 for wire antennas that have rotational symmetry in the azimuthal direction, such as a monopole element with a *uniform radial-wire groundscreen* [6, 7]. Version NEC-3I extends NEC-3 to include the case of insulated wires [8, 9].

The NEC-2 program is available to the public, whereas the NEC-3, NEC-GS, and NEC-3I programs are presently available only to U. S. Department of Defense contractors after completion and approval of a NEC order form obtainable from LLNL.

Code documentation has been produced by LLNL for the NEC-2 version and, in a more limited form, for the NEC-3, NEC-GS, and NEC-3I versions. The NEC-2 documentation consists of the theory and code in volume 1 of reference 1 and a user's guide in volume 2 of reference 1. The NEC-3, NEC-GS, and NEC-3I documentations are in the form of user's guide supplements given in references 5, 7, and 8, respectively. The NEC-2 user's guide and NEC-3 and NEC-GS user's guide supplements give examples of input and output files for most of the options available. Sample input and output files for the NEC-3I program are given in reference 9.

Code validation efforts by LLNL, for antennas near ground, are summarized in reference 10. In addition to several internal consistency checks, reference 10 compares NEC results with those from theoretical models, numerical codes, and to a lesser extent with actual measurements.

The present paper reports recent MITRE efforts to provide additional validation of the NEC-3, NEC-GS, and NEC-3I programs. Validation results by MITRE are described in sections 2, 3, 4, and 5 for NEC-3 in the Fresnel reflection coefficient option, NEC-3 in the Sommerfeld integral option, NEC-GS, and NEC-3I programs, respectively.

SECTION 2

VERSION NEC-3, FRESNEL REFLECTION COEFFICIENT OPTION

2.1 CODE CORRECTIONS

2.1.1 NEC-3 Error Alert

LLNL has documented an error that occurs in the NEC-3 and NEC-3I (but not NEC-2) codes in the Fresnel reflection coefficient option. The error occurs when calculating the radiation pattern for an antenna with a radical-wire ground screen [11]. The corrected statements are given in reference 11.

2.1.2 Selection of Square Root Branch

The NEC-2 and NEC-3 codes, in the Fresnel reflection coefficient option, select the principal value branch of each square root occurring in the equations for the Fresnel reflection coefficients. The question arises as to whether the principal value is the correct branch of the square root, particularly in cases where the effective complex permittivity of the ground plane at the air-ground plane interface has a negative real part. Such cases can occur for wire grids, in free space or in proximity to earth, because the ground plane permeability is conventionally set equal to that of free space.

The Fresnel reflection coefficients R_V and R_H , for vertical and horizontal polarizations, respectively, are given by equations (179) and (180) in Volume 1 of reference 1 as

$$R_V = \frac{\cos \phi - Z_R (1 - Z_R^2 \sin^2 \theta)^{1/2}}{\cos \phi + Z_R (1 - Z_R^2 \sin^2 \theta)^{1/2}} \quad (2-1)$$

$$R_H = \frac{(1 - Z_R^2 \sin^2 \theta)^{1/2} - Z_R \cos \theta}{(1 - Z_R^2 \sin^2 \theta)^{1/2} + Z_R \cos \theta} \quad (2-2)$$

Our investigation concludes that the principal value is the correct branch of the quantities $Z_R \equiv [(\epsilon_1 / \epsilon_0) - j(\sigma_1 / \omega \epsilon_0)]^{-1/2}$ and $(1 - Z_R^2 \sin^2 \theta)^{1/2} / Z_R$ in equations (2-1) and (2-2) regardless of whether the effective dielectric constant ϵ_1 / ϵ_0 is negative, assuming a passive ground medium $[(\sigma_1 / \omega \epsilon_0) \geq 0]$. This conclusion follows from the requirements that the complex wave number $k = \omega(\epsilon_0 \mu_0)^{1/2} / Z_R$ have an argument in the fourth quadrant of the complex plane for a plane wave propagating with a time dependence of the form

$$E = E_0 \exp \left[j \left(\omega t - \vec{k} \cdot \vec{r} \right) \right]$$

and that the magnitude of the Fresnel reflection coefficient does not exceed unity for a plane wave incident from a lossless medium onto a passive medium [11, 13].

In equations (2-1) and (2-2) the principal values of $(1 - Z_R^2 \sin^2 \theta)^{1/2}$ each satisfy the condition $|R_V| \leq 1$ for the case of a plane wave incident from a lossless medium onto a passive medium, regardless of whether $\text{Re}(1/Z_R^2) = \epsilon_1 / \epsilon_0$ is positive or negative. Equations (2-1) and (2-2) are in the same form as that given by Stratton [14].

If one divides the numerator and denominator of equation (2-1) by Z_R^2 , one obtains the form given by Reed and Russell [15], namely,

$$R_V = \frac{(1/Z_R)^2 \cos \theta - [(1/Z_R)^2 - \sin^2 \theta]^{1/2}}{(1/Z_R)^2 \cos \theta + [(1/Z_R)^2 - \sin^2 \theta]^{1/2}} \quad (2-3)$$

In equation (2-3), the principal value of $[(1/Z_R)^2 - \sin^2 \theta]^{1/2}$ satisfies the condition $|R_V| \leq 1$ for the case of a plane wave incident from a lossless medium onto a passive medium only if $\text{Re}(1/Z_R^2) = \epsilon_1 / \epsilon_0 \geq 0$. For $\epsilon_1 / \epsilon_0 < 0$, the condition $|R_V| \leq 1$ is satisfied by the nonprincipal value of $[(1/Z_R)^2 - \sin^2 \theta]^{1/2}$.

The form of the reflection coefficient given by equation (2-1), unlike that given by equation (2-3), gives correct results for all cases of a wave incident from a lossless medium onto a passive medium if the square roots are restricted to their principal values. The validity, of the principal values for the square roots in equations (2-1) and (2-2), subject to this condition, has been confirmed by Burke [16]. In reference 16, please note that a wire grid in free space has a relative permittivity ($=$ principal value $\left[(Y_g + Y_o) / Y_o \right]^2$) whose imaginary part has a conductivity greater than zero and whose real part (the dielectric constant) is negative. The wire grid admittance is given by $Y_g \approx -jY_o / [(s/\lambda) \ln(s/\pi d)]$ and the free space admittance is given by $Y_o = (\epsilon_o / \mu_o)^{1/2}$ where s is the grid spacing ($s/\lambda \ll 1$) and d is the wire diameter ($d/s \ll 1$). However, even for the case of a negative dielectric constant and positive conductivity, the principal values of the square roots yield valid results. For that case, $|R_v| = 1$ and $\arg R_v$ differs by 180 degrees from that for a negative dielectric constant and positive conductivity. This result is similar to the case of a perfect ground plane for which $|R_v| = 1$ and $\arg R_v$ differs by 180 degrees from that for an imperfect ground plane at an angle of incidence equal to 90 degrees.

2.2 COMPARISON WITH OTHER MODELS

Fresnel reflection coefficient models for antennas in proximity to earth are generally grossly inaccurate in determining input impedance, radiation efficiency, and power gains unless the ground plane and monopole element current distributions are predetermined by other methods such as the method of moments. However, Fresnel reflection coefficient models are accurate when determining the absolute directive gain or directive gain pattern for the case of an antenna element in proximity to earth, or a ground plane of infinite extent (see discussion at end of this section). These remarks are applicable not only to the Fresnel reflection option of the NEC-2 and NEC-3 programs, but also to any model which attempts to approximate the ground plane current, originating from a spherical wave source, by that determined from a plane-wave, Fresnel reflection coefficient model.

The antenna element current distribution, in the reflection coefficient option of NEC-3, is determined by considering the mutual impedance between the source antenna element and its ground plane image. The ground plane image is determined by considering the Fresnel reflection coefficient only for the ground plane (or earth) directly below the antenna element. Consequently, ground screens of small density or extent will yield the same reflection coefficient as a ground screen of large density or extent. Furthermore, the Fresnel reflection coefficient model neglects groundscreen edge diffraction and underestimates earth losses, both of which can be significant for small ground planes. For these reasons, the input impedance of an antenna element in proximity to earth is poorly estimated by the reflection coefficient option unless the element has a ground plane of sufficiently large density and extent.

Fresnel reflection coefficient models greatly overestimate the radiation efficiency of antennas in close proximity to earth because such models only consider ground losses caused by plane-wave reflection and refraction and ignore spherical-wave generation of a leaky evanescent surface wave that is generated in the air medium in proximity to the air-earth interface. The surface wave, with an evanescent field in the air-medium only, leaks energy into the earth medium but not into the air medium [17, 18]. A comparison of the radiation efficiency calculated by the Sommerfeld option of NEC-3 (which considers surface wave ground losses) with that calculated by a Fresnel reflection coefficient model is shown in table 1 at a frequency of 15 MHz for a vertically polarized monopole element resting on earth. For medium dry earth, the Sommerfeld option yields numeric radiation efficiencies of 0.008 and 0.163 for element lengths of 0.05 and 0.25 wavelengths, respectively, whereas the Fresnel reflection coefficient model predicts radiation efficiencies of 0.316 and 0.305, respectively. In this example, the Fresnel reflection coefficient model overestimates the radiation efficiency by 4000 percent and 61 percent for element lengths of 0.05 and 0.25 wavelengths, respectively. The Fresnel reflection coefficient model is therefore inappropriate for computing radiation efficiency for antennas in close proximity to earth. The error in using the Sommerfeld integral option is no more than 23 per cent for the worse case, as discussed in section 3.2.2.

Table 1. Radiation Efficiency of a Vertically Polarized Thin Monopole Element of Length h Whose Base is at Zero Height Above Earth, $f = 15$ MHz

CCIR-527-1 Earth Classification	Radiation Efficiency η (numeric)					
	$h/\lambda = 0.01$	$h/\lambda = 0.05$	$h/\lambda = 0.10$	$h/\lambda = 0.15$	$h/\lambda = 0.20$	$h/\lambda = 0.25$
(1) Perfect Ground						
Sommerfeld	1.000	1.000	1.000	1.000	1.000	1.000
Fresnel	1.000	1.000	1.000	1.000	1.000	1.000
(2) Sea Water						
Sommerfeld*	0.005	0.317	0.660	0.752	0.798	0.823
Fresnel	0.894	0.891	0.889	0.886	0.882	0.877
(3) Fresh Water						
Sommerfeld*	0.000	0.014	0.083	0.146	0.211	0.273
Fresnel	0.525	0.520	0.517	0.511	0.504	0.496
(4) Wet Ground						
Sommerfeld*	0.000	0.005	0.033	0.063	0.101	0.144
Fresnel	0.407	0.402	0.399	0.393	0.387	0.379
(5) Medium Dry Ground						
Sommerfeld*	0.000	0.008	0.050	0.087	0.125	0.163
Fresnel	0.318	0.316	0.315	0.313	0.309	0.305
(6) Very Dry Ground						
Sommerfeld*	0.000	0.003	0.020	0.039	0.063	0.090
Fresnel	0.175	0.175	0.175	0.175	0.176	0.178
(7) Pure Water, 20 °C						
Sommerfeld*	0.002	0.096	0.216	0.282	0.334	0.375
Fresnel	0.510	0.510	0.508	0.505	0.501	0.495
(8) Ice, -1 °C						
Sommerfeld*	0.000	0.003	0.020	0.039	0.063	0.090
Fresnel	0.175	0.175	0.175	0.176	0.176	0.178
(9) Ice, -10 °C						
Sommerfeld*	0.000	0.008	0.046	0.077	0.107	0.135
Fresnel	0.175	0.175	0.175	0.176	0.177	0.179
(10) Average Land (TCI)						
Sommerfeld*	0.000	0.001	0.007	0.016	0.027	0.044
Fresnel	0.294	0.287	0.282	0.277	0.270	0.262
(11) Free Space						
Sommerfeld	1.000	1.000	1.000	1.000	1.000	1.000
Fresnel	1.000	1.000	1.000	1.000	1.000	1.000

* More accurate result

Sommerfeld = NEC-3 program in Sommerfeld option with voltage excitation source between earth and the base of the element ($b/\lambda = 10^{-5}$)

Fresnel = Program MODIFIED IMAGES assuming Fresnel reflection coefficient and sinusoidal current distribution on elements

Despite the inadequacy of Fresnel reflection coefficient models for estimating the input impedance and radiation efficiency of antenna elements in close proximity to earth, such models are accurate in estimating the antenna's absolute directive gain and directive gain pattern for the case of an antenna element in proximity to earth (or to a ground plane of infinite extent) and for an antenna element whose ground plane current distribution is pre-determined by other methods. The directive gains, computed by the NEC-3 Sommerfeld option, Richmond's method of moments, and a Fresnel reflection coefficient model, are compared in table 2 for the case of a vertically polarized quarter-wave monopole element on medium dry earth. Each model gives the same directive gain to within 0.04 dB at a given angle of incidence.

However, even for a case where directive gain is correctly given by a Fresnel reflection coefficient model, the power gain (= directive gain x radiation efficiency) is incorrectly given because the radiation efficiency is grossly overestimated by the Fresnel reflection model.

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Table 2. Directive Gain of a Thin, Vertically Polarized Quarter-wave Monopole Element Resting on Medium Dry Earth, 15 MHz

Angle of Incidence, θ (deg)	Directive Gain (dBi)			Angle of Incidence, θ (deg)	Directive Gain (dBi)		
	NEC-3 (Sommerfeld Option) ¹	Richmond (Method of Moments) ²	Modified Images (Fresnel Coefficient) ³		NEC-3 (Sommerfeld Option) ¹	Richmond (Method of Moments) ²	Modified Images (Fresnel Coefficient) ³
0	-∞	-∞	-∞	46	3.80	3.78	3.80
2	-22.90	-22.88	-22.88	48	4.07	4.05	4.07
4	-16.88	-16.86	-16.86	50	4.31	4.29	4.31
6	-13.35	-13.34	-13.34	52	4.53	4.50	4.53
8	-10.86	-10.85	-10.84	54	4.71	4.69	4.71
10	-8.92	-8.91	-8.91	56	4.86	4.84	4.86
12	-7.34	-7.33	-7.33	58	4.98	4.96	4.98
14	-6.00	-5.99	-5.99	60	5.07	5.04	5.06
16	-4.85	-4.84	-4.84	62	5.11	5.08	5.11
18	-3.83	-3.82	-3.82	64	5.12	5.09	5.11
20	-2.92	-2.92	-2.91	66	5.07	5.04	5.07
22	-2.10	-2.10	-2.09	68	4.98	4.95	4.98
24	-1.36	-1.36	-1.35	70	4.82	4.79	4.82
26	-0.68	-0.68	-0.67	72	4.59	4.56	4.59
28	-0.06	-0.06	-0.05	74	4.27	4.23	4.26
30	0.52	0.52	0.53	76	3.83	3.80	3.83
32	1.05	1.05	1.06	78	3.24	3.21	3.24
34	1.54	1.54	1.55	80	2.44	2.40	2.44
36	2.00	1.99	2.00	82	1.33	1.29	1.32
38	2.42	2.41	2.42	84	-0.29	-0.33	-0.29
40	2.81	2.80	2.81	86	-2.86	-2.90	-2.87
42	3.17	3.16	3.17	88	-7.85	-7.89	-7.85
44	3.50	3.48	3.50	90	-∞	-∞	-∞

1. Voltage excitation source between the earth and the base of the element.

2. Program RICI4MD4 with small ground plane of normalized radius $2\pi a/\lambda = 0.025$ wave numbers.

3. Program MODIFIED IMAGES assumes Fresnel reflection coefficient and sinusoidal current distribution on element.

SECTION 3

VERSION NEC-3, SOMMERFELD INTEGRAL OPTION

3.1 CODE EXTENSIONS

3.1.1 Incorporation of Program SOMNTX

The Sommerfeld integral option of the NEC-3 code requires an input file (TAPE 21) generated in advance by the auxiliary program SOMNTX [5]. SOMNTX computes the Sommerfeld integral by requiring only three input parameters: the relative dielectric constant of the ground (ϵ_r), the conductivity σ of the ground in mhos/m, and the RF frequency f in MHz. A drawback of the program SOMNTX, as implemented by LLNL, is that it requires separate files for each frequency and Earth complex permittivity because only one set of input parameters (ϵ_r , σ , f) can be entered into a file at a time.

Several years ago, MITRE incorporated SOMNTX into NEC-3 so that the separate input file TAPE 21 is not required. The program is implemented for operation on a FPS-M64 computer with 64 bit precision. Computer runs of test cases that were supplied with the NEC-3 program yielded numerical results before and after the code modification that agreed to five significant figures.

3.1.2 Increase of Maximum Matrix Size

The maximum matrix size, as implemented by LLNL, limits the number of unknown current variables (or wire segments) to 300 for the NEC-3 version. MITRE extended the maximum matrix size by a factor of five (to 1500) for the NEC-3 version.

3.2 COMPARISON WITH OTHER MODELS AND MEASUREMENTS

3.2.1 LLNL Validation Efforts

LLNL has compared numerical results for the input impedance and electric field of a sloping base long-wire antenna over conducting Earth, obtained from NEC-3 in the

Sommerfeld integral option, with measurements by Breakall and Christman [10]. Predicted versus measured values differed approximately by 25 to 100 percent for input resistance, ± 30 ohms about 0 ohms for input reactance, and 1 to 9 dB μ V/m for the electric field.

3.2.2 Modified Radiation Efficiency of a Vertically Polarized, Hertzian Dipole in Proximity to Dielectric Earth

NEC-3 results by Burke [20], for the modified radiation efficiency η_d (defined in figure 1) of an electrically short, vertical dipole above dielectric Earth, are compared in figures 1 and 2 with King's analytical results for a Hertzian vertical dipole [18] obtained by integrating the vertical component of the Poynting vector along a far-field line parallel to the air-Earth interface. The two models give similar results for sufficiently large values of the Earth dielectric constant, but differ by 15 percent for the Earth dielectric constant $\epsilon_r = 9$ (or $k_1/k_2 = 3$) when the dipole is at zero height above the Earth. The results of King are approximate because his analytical model is subject to the condition $\epsilon_r \gg 1$ (or equivalently $k_1/k_2 \geq 3$) except for the condition $\epsilon_r = 1$ which is treated separately. The NEC-3 results are for a vertical dipole of half-length = 10^{-4} wavelengths and radius = 10^{-6} wavelengths at a height $|z_0|$ above earth measured from the center-feed of the dipole. In this paper the convention is followed that lower-case z designates the Earth's vertical position with respect to the antenna ground plane (or base of the antenna element, in the absence of a ground plane) and upper-case Z designates impedance.

The case of a lossless antenna element over dielectric Earth provides an excellent opportunity for testing the accuracy of the antenna input current, I , computed by the Sommerfeld integral option of the NEC-3 code. The antenna power gain \bar{G} averaged over the radiation sphere (solid angle of 4π steradians) is defined as

$$\bar{G} = P_{rad} / P_{in} = (P_{air} + P_{earth}) / P_{in} \quad (3-1)$$

where

$$\begin{aligned} P_{rad} &= \text{total far-field radiated power} = P_{air} + P_{earth} \\ P_{air} &= \text{far-field radiated power in the air} \end{aligned}$$

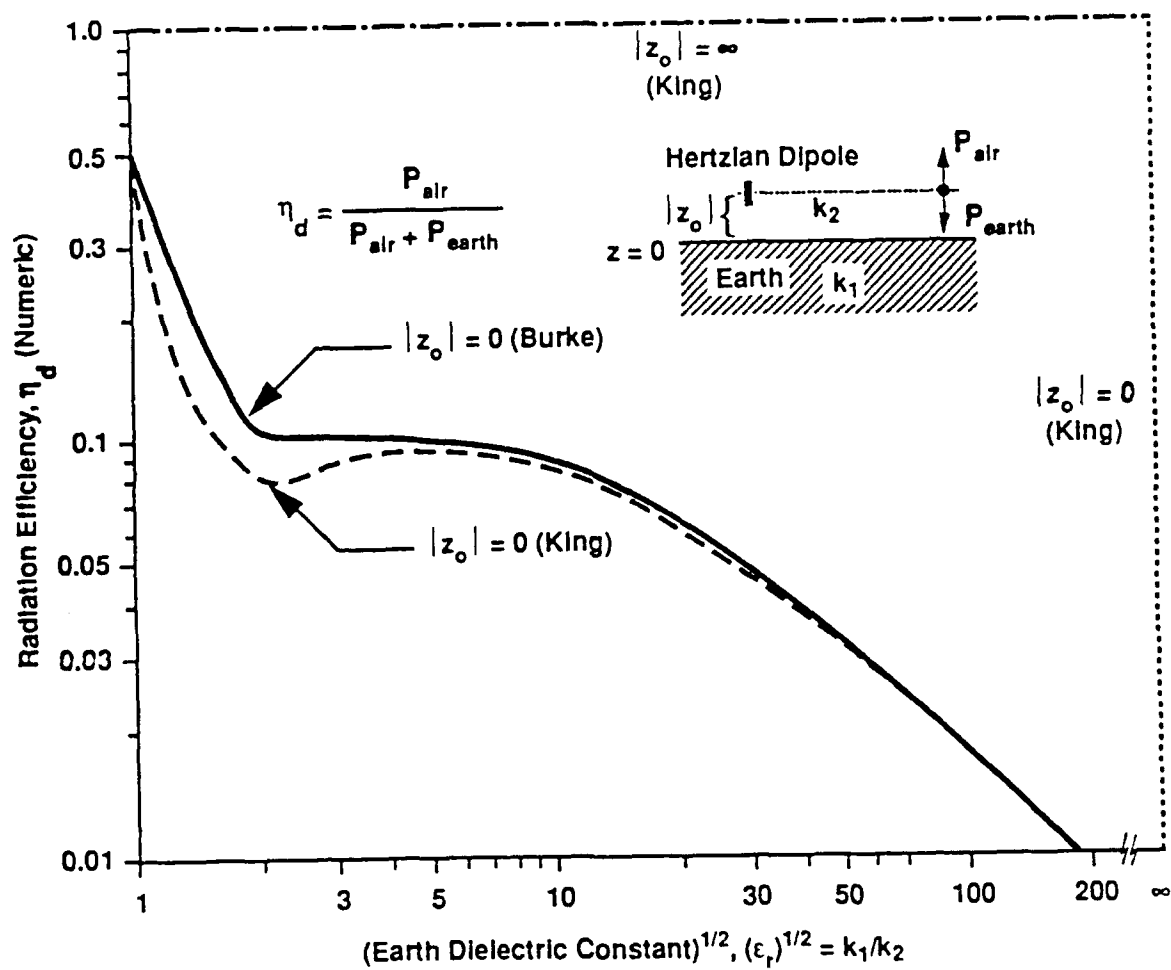


Figure 1. Radiation Efficiency of a Vertical Hertzian Dipole at Zero Height Above Dielectric Earth

Hertzian dipole at height $|z_0|$ in air (k_2)
over a dielectric half-space ($k_1 = k_2 \sqrt{\epsilon_r}$)

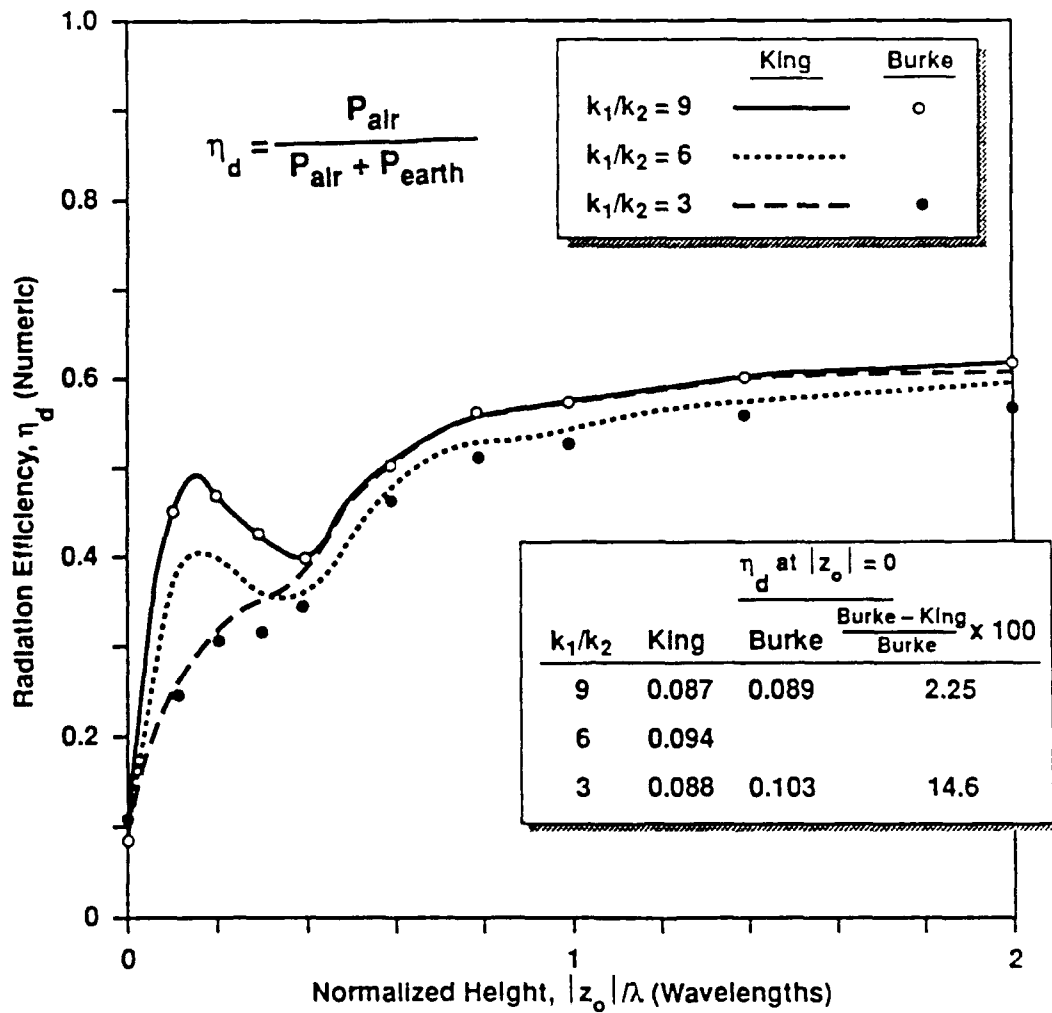


Figure 2. Radiation Efficiency of a Vertical Hertzian Dipole at Various Heights Above Dielectric Earth

- P_{earth} = far-field radiated power in dielectric Earth
 P_{in} = input power delivered to the antenna = $(1/2) \text{Re}(VI^*)$
 V = input voltage complex amplitude (set equal to 1 volt) in the NEC program for a steady-state sinusoidal source.
 I^* = conjugate input current complex amplitude (amperes) that is solved for in the NEC-3 program. The asterisk denotes "conjugate."

The quantities P_{air} and P_{earth} , for an antenna element with azimuthal symmetry, is given by

$$P_{air} = [r^2 / (2Z_0)] 2\pi \int_0^{\pi/2} (\vec{E} \cdot \vec{E}^*) \sin \theta \, d\theta \quad (3-2)$$

$$P_{earth} = [r^2 / (2Z_0)] 2\pi \int_{\pi}^{\pi/2} (\vec{E} \cdot \vec{E}^*) \sin \theta \, d\theta \quad (3-3)$$

where

- r = distance from the antenna element to the far-field point $P(r, \theta, \phi)$ (m)
 E = electric field intensity at the far-field point $P(r, \theta, \phi)$ = (V/m)
 $Z_0 = (\mu_0 / \epsilon_0)^{1/2}$ = free space wave impedance (ohms)

For a lossless antenna over dielectric Earth, the average power gain \bar{G} equals 1, if there are no errors in the NEC-3 program and the computer has infinite precision. Assuming that the computer has sufficient precision and that the integration steps in equations (3-2) and (3-3) are sufficiently small, then any deviation of \bar{G} from unity is a measure of the accuracy of the current I computed by the NEC-3 program. The reason is that P_{in} is proportional to I whereas, P_{air} and P_{earth} are proportional to $|I|^2$ (because E is proportional to I).

The quantity \bar{G} , as computed by the NEC-3 program, is a weak function of the number N of segments (or current variables) chosen to represent the antenna element. Whenever one uses the method-of-moments, too coarse a segmentation results in poor accuracy due to undersampling the current distribution. Too fine a segmentation can again result in poor accuracy because of round-off errors caused by the finite precision of the computer. The

element segmentation, for vertical dipole and monopole elements above Earth, is shown for a voltage excitation source in figure 3. For a thin, electrically short dipole at a height $|z_0|/\lambda = 0.4$ above dielectric Earth ($\epsilon_r = 9, \sigma = 0$), \bar{G} differs from unity by 1.4 percent for $N=5$ and 0.1 percent for $N = 101$ (see table 3). For the same dipole at a height $|z_0|/\lambda = 0.0001$ above the same dielectric Earth, \bar{G} differs from unity by 22.3 percent for $N = 11$ and 22.6 percent for $N = 101$. Even though the element segment length for $N = 101$ in table 3 is one-half the recommended minimum segment length relative to the segment radius (see section 4.3.2), the results are not significantly different than for $N = 51$.

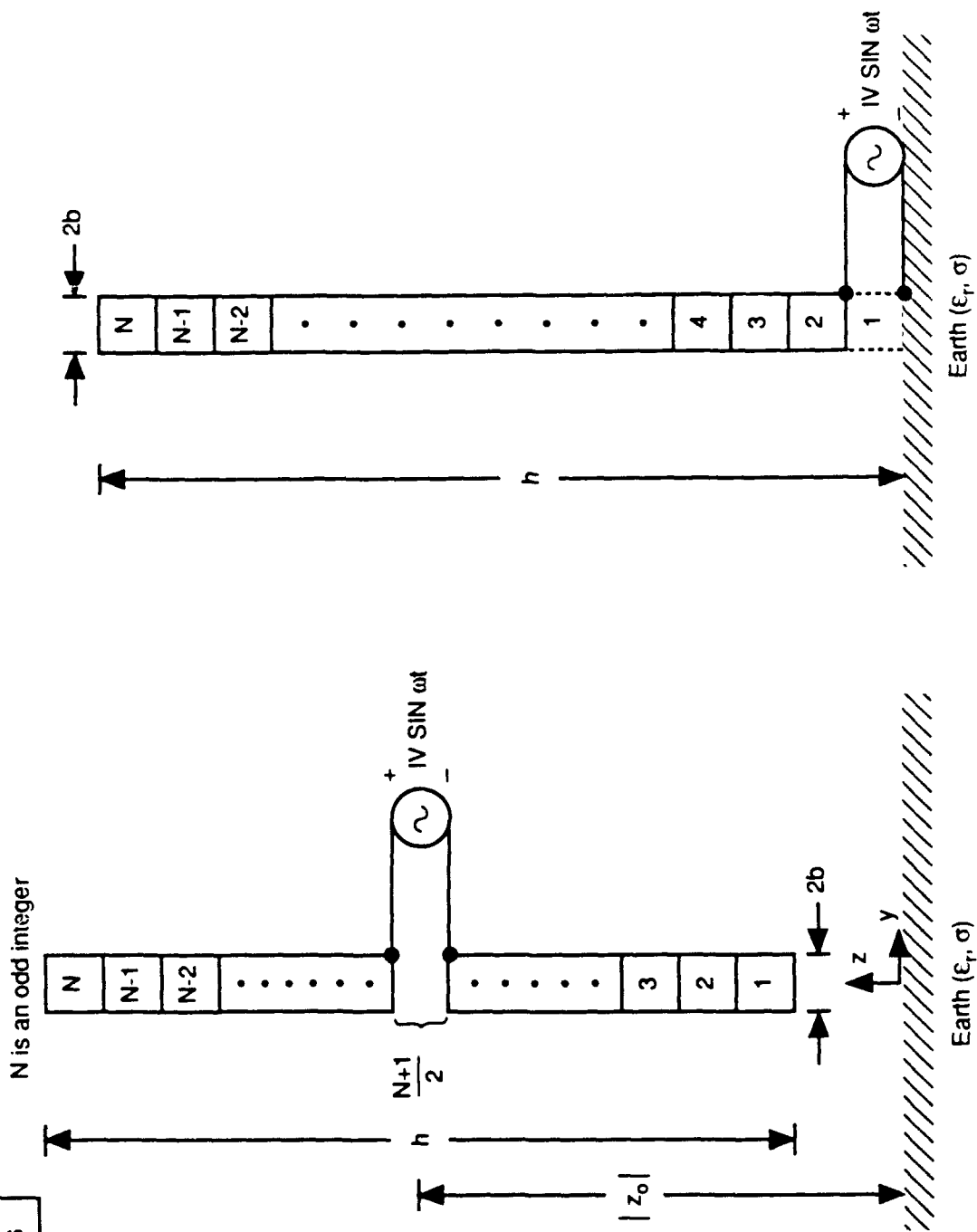
The difference of \bar{G} from unity increases with increasing Earth dielectric constant and decreasing element height above earth. For a dipole at a height $|z_0|/\lambda = 0.0001$, \bar{G} differs from unity by 7.6 percent for $\epsilon_r = 2.25$ and 40.2 percent for $\epsilon_r = 81$ (see table 4). For a dipole above dielectric Earth with $\epsilon_r = 9$, \bar{G} differs from unity by 22.6 percent for $|z_0|/\lambda = 0.0001$ and 0.7 percent for $|z_0|/\lambda = 2.0$ (see table 5).

The differences of \bar{G} from unity in tables 3 through 6 indicate that the NEC-3 program has inaccuracies as much as 25 percent or more in computing input current, input impedance, and input power for an electrically short antenna element in close proximity to Earth. These inaccuracies do not apply to the computation of modified efficiency η_d , but would also affect the computation of radiation efficiency if the antenna element were in proximity to lossy Earth since radiation efficiency is a function of the absolute accuracy of the input current.

For dielectric Earth the modified radiation efficiency $\eta_d = \eta/\bar{G} = P_{air}/(P_{air} + P_{earth})$ is not dependent upon the absolute accuracy of the input current since both P_{air} and P_{earth} are proportional to the same computed value of input current. Therefore, the modified radiation efficiency computed by NEC-3 for dielectric Earth is accurate to within the precision of the computer and the size of the integration steps $\Delta\theta$ of the far-field power density. In NEC-3, the modified radiation efficiency η_d is computed for dielectric Earth as the quotient of P_{air} divided by $(P_{air} + P_{earth})$, namely:

$$\eta_d = \text{modified validation efficiency} = \eta / \bar{G} = P_{air} / (P_{air} + P_{earth}) \quad (3-4)$$

IY0006



(a) Dipole with Voltage Source Excitation (b) Monopole with Voltage Source Excitation

Figure 3. Segmentation of Vertical Antenna Element Above Earth

IY0010

Table 3. Effect of Number of Dipole Segments on Average Power Gain and Radiation Efficiency Computed by Program NEC-3 for an Electrically-Small, Vertical Dipole at Heights $|z_0/\lambda| = 0.4$ and 0.0001 above Dielectric Earth ($\epsilon_r = 9$, $\sigma = 0$)

No. of Seg-ments, N	*Average Power Gain, $\bar{G} = (P_{air} + P_{earth})/P_{in}$		$P_{air}/P_{in}, \eta$		Radiation Efficiency, $\eta_d = P_{air}/(P_{air} + P_{earth}) = \eta / \bar{G}$	
	$ z_0/\lambda = 0.4$	$ z_0/\lambda = 0.0001$	$ z_0/\lambda = 0.4$	$ z_0/\lambda = 0.0001$	$ z_0/\lambda = 0.4$	$ z_0/\lambda = 0.0001$
5	0.9858	—	0.3356	—	0.3404	—
11	0.9963	1.223	0.3392	0.1260	0.3404	0.1030
21	0.9983	—	0.3399	—	0.3404	—
31	0.9989	1.226	0.3400	0.1263	0.3404	0.1030
41	0.9989	—	0.3400	—	0.3404	—
51	0.9989	—	0.3401	—	0.3404	—
81	0.9990	—	0.3401	—	0.3404	—
101	0.9990	1.226	0.3401	0.1263	0.3404	0.1030

Dipole length $h/\lambda = 2 \times 10^{-4}$

Dipole radius $b/\lambda = 1 \times 10^{-6}$

Integration step $\Delta\theta = 1.0$ deg, $0 \leq \theta \leq 90$ deg; 0.1 deg, $90 \leq \theta \leq 180$ deg

P_{air}, P_{earth} = far-field radiated powers in air and dielectric earth, respectively

$P_{in} = (1/2) \text{Re}(VI^*) = (1/2) IV \text{Re } I^*$

* $\bar{G} = 1$, for a loss-less element over dielectric earth, if there were no errors in NEC-3 program and the computer had infinite precision.

IY0011

Table 4. Effect of Earth Dielectric Constant on Average Power Gain and Radiation Efficiency Computed by Program NEC-3 for an Electrically-Small, Vertical Dipole at Height $|z_0|/\lambda = 0.0001$ above Dielectric Earth ($\sigma = 0$)

Dielectric Constant, ϵ_r	*Average Power Gain, \bar{G} = $(P_{air} + P_{earth})/P_{in}$	$P_{air}/P_{in}, \eta$	Radiation Efficiency, η_d = $P_{air}/(P_{air} + P_{earth})$ = η / \bar{G}
1.0	0.9997	0.4998	0.5000
2.25	1.0763	0.1400	0.1387
4.0	1.1301	0.1272	0.1126
9.0	1.2257	0.1263	0.1030
16.0	1.2826	0.1300	0.1014
25.0	1.3096	0.1311	0.1001
36.0	1.3442	0.1308	0.0973
49.0	1.356	0.1294	0.0954
64.0	1.3848	0.1272	0.0919
81.0	1.4017	0.1244	0.0888
100.0	1.4026	0.1214	0.0866
400.0	1.4840	0.0919	0.0620
900.0	1.5258	0.0724	0.0475
1600.0	1.5295	0.0593	0.0388
2500.0	1.5321	0.0501	0.0327
3600.0	1.5308	0.0433	0.0283
4900.0	1.5278	0.0382	0.0250
6400.0	1.5223	0.0342	0.0225
8100.0	1.5148	0.0309	0.0204

Dipole length $h/\lambda = 2 \times 10^{-4}$, dipole radius $b/\lambda = 1 \times 10^{-6}$, no. of dipole segments $N = 31$

Integration step $\Delta\theta = 1.0$ deg, $0 \leq \theta \leq 90$ deg; 0.1 deg, $90 < \theta \leq 180$ deg.

* $\bar{G} = 1$, for a loss-less element over dielectric earth, if there were no errors in NEC-3 program and the computer had infinite precision

IY0012

Table 5. Effect of Earth Dipole Height on Average Power Gain and Radiation Efficiency Computed by Program NEC-3 for an Electrically-Small, Vertical Dipole above Dielectric Earth ($\epsilon_r = 9$, $\sigma = 0$)

Height Above Earth, $ z_0 /\lambda$	*Average Power Gain, \bar{G} $= (P_{air} + P_{earth})/P_{in}$	P_{air}/P_{in} , η	Radiation Efficiency, η_d $= P_{air}/(P_{air} + P_{earth})$ $= \eta / \bar{G}$
0.0001	1.2257	0.1263	0.1030
0.0003	1.2257	0.1266	0.1033
0.001	1.2165	0.1269	0.1043
0.003	1.1895	0.1276	0.1073
0.01	1.1053	0.1301	0.1177
0.03	1.0096	0.1498	0.1483
0.1	0.9988	0.2410	0.2413
0.2	1.0007	0.2971	0.2969
0.3	0.9987	0.3078	0.3082
0.4	0.9987	0.3400	0.3404
0.6	0.9970	0.4609	0.4623
0.8	0.9971	0.5082	0.5097
1.0	0.9958	0.5207	0.5229
1.4	0.9945	0.5463	0.5493
2.0	0.9928	0.5593	0.5633

Number of dipole segments $N = 31$

Dipole length $h/\lambda = 2 \times 10^{-4}$

Dipole radius $b/\lambda = 1 \times 10^{-6}$

Integration step $\Delta\theta = 1.0$ deg, $0 \leq \theta \leq 90$ deg; 0.1 deg, $90 < \theta < 180$ deg

* $\bar{G} = 1$, for a loss-less element over dielectric earth, if there were no errors in NEC-3 program and the computer had infinite precision

IY0013

Table 6. Effect of Number of Dipole Segments on Average Power Gain and Radiation Efficiency Computed by Program NEC-3 for an Electrically-Small, Vertical Monopole Whose Base Rests on Dielectric Earth ($\epsilon_r = 9$, $\sigma = 0$)

No. of Segments, N	*Average Power Gain, \bar{G} $= (P_{air} + P_{earth})/P_{in}$	$P_{air}/P_{in}, \eta$	Radiation Efficiency, η_d $= P_{air}/(P_{air} + P_{earth})$ $= \eta / \bar{G}$
5	1.1703	0.1205	0.1030
11	1.1871	0.1223	0.1030
21	1.1877	0.1223	0.1030
31	1.1837	0.1219	0.1030

Monopole length $h/\lambda = 2 \times 10^{-4}$

Monopole radius $b/\lambda = 1 \times 10^{-6}$

Integration step $\Delta\theta = 1.0$ deg, $0 \leq \theta \leq 90$ deg; 0.1 deg, $90 < \theta \leq 180$ deg.

* $G = 1$, for a loss-less element over dielectric earth if there were no error in NEC-3 program and the computer had infinite precision.

where

$$\eta = \text{radiation efficiency} = P_{\text{air}}/P_{\text{in}}$$

$$\bar{G} = (P_{\text{air}} + P_{\text{earth}}) / P_{\text{in}} \text{ [see equation (3-1)]}$$

The modified radiation efficiency η_d computed by NEC-3 is shown in tables 3 through 5 for a dipole above dielectric earth. In table 3, the radiation efficiency is independent of the number of element segments as is also the case for an electrically-small vertical monopole element whose base rests on earth (see table 6). The modified radiation efficiencies of an electrically-small vertical dipole and monopole, of the same length and radius and whose bases rest on earth, should be identical. This result is achieved by the NEC-3 program (compare table 3 for $|z_0|/\lambda = 0.0001$ with table 6). If the monopole element in table 6 is increased to a quarter-wave length with 25 segments, the average power gain $\bar{G} = 0.9990$ [22].

3.2.3 Radiation Resistance, Radiation Efficiency, and Directive Gain of a Vertical Quarter-Wave, Monopole Element in Close Proximity to Lossy Earth

In the preceding subsection, NEC-3 results were evaluated for an antenna element in proximity to dielectric earth. This subsection evaluates NEC-3 results for an antenna element in proximity to lossy earth.

NEC-3 results of directive gain, for thin, vertical, quarter-wave element on medium-dry earth at 15 MHz are in close agreement (within 0.04 dBi) with results from Richmond's method of moments model and also from a Program MODIFIED IMAGES, a Fresnel reflection coefficient model developed by MITRE (see table 2). The reason for the close agreement is that the directive gain does not depend upon the absolute accuracy of the antenna input current.

The radiation resistance and radiation efficiency of an antenna element in proximity to lossy earth does depend upon the absolute accuracy of the antenna input current. NEC-3 results of radiation resistance and radiation efficiency at 15 MHz, for a thin, vertical, quarter-wave element on lossy earth with no ground plane ($2\pi a/\lambda = 0$), are compared in table 7 with

IY0014

Table 7. Radiation Resistance and Efficiency of a Vertical Quarter-Wave, Monopole Element on Flat Earth; $f = 15$ MHz, $b/\lambda = 1.0 \times 10^{-6}$

Earth Classification (ϵ_r, σ S/m)	Radiation Resistance (Ohms)			Radiation Efficiency (Numeric)		
	*NEC-3	**RICHMD4	***Percent Difference	*NEC-3	**RICHMD4	***Percent Difference
Sea water (70, 5)	34.0	29.5	15.0	0.823	0.799	29.4
Fresh water (80, 3.0×10^{-2})	19.1	17.3	10.4	0.273	0.347	34.3
Wet ground (30, 1.0×10^{-2})	14.5	13.2	10.3	0.144	0.229	36.9
Medium dry ground (15, 1.0×10^{-3})	11.5	10.5	10.3	0.163	0.210	22.7
Very dry ground (3, 1.0×10^{-4})	6.2	5.7	9.6	0.091	0.145	37.6
Pure water, 20°C (80, 1.7×10^{-3})	19.1	17.3	9.4	0.375	0.378	0.8
Ice (-1°C) (3, 9.0×10^{-5})	6.2	5.7	9.6	0.091	0.148	38.8
Ice (-10°C) (3, 2.7×10^{-5})	6.2	5.7	9.5	0.136	0.171	20.8
Average land (10, 5.0×10^{-3})	9.9	9.0	10.3	0.044	0.105	58.3

* Number of element segments, $N = 25$; voltage source excitation at $N = 1$

** Disk ground plane radius, $2\pi a/\lambda = 0.025$ wavenumbers

*** $|(\text{NEC-3} - \text{RICHMD4}) / \text{RICHMD4}| \times 100$

numerical results from Richmond's method-of-moments program RICHMOND4 [19, 21] for a quarter-wave element on a disk ground plane of radius $2\pi a/\lambda = 0.025$ wave-number that rests on lossy earth. The results differ by approximately 10 percent for radiation resistance and by more than 25 percent for radiation efficiency. These differences are attributable to the difference in base charge density by a factor of 4,000 caused by the different configurations of the two models [22]. In NEC-3, the current produced by the charge distribution is discharged into the Earth through an element of radius 10^{-6} wavelengths, whereas in RICHMOND4 the current is discharged into the Earth through a ground plane of radius 4×10^{-3} wavelengths. The effect of base charge density on radiation efficiency is verified by noting that NEC-3 results for radiation efficiency for a quarter-wave monopole element on medium dry Earth increases from 0.163 to 0.206 (comparable to the RICHMOND4 result of 0.210) if a 128 radial-wire ground plane of radius 0.01 wavelengths augments the monopole element [33]. Increasing the number of monopole segments from 4 to 20 in RICHMOND4 has no significant effect in modifying the table 7 results for radiation efficiency.

3.2.4 Propagation Constant of Current on Bare, Horizontal Wire (Beverage Antenna) Above Lossy Earth

Recent measurements of the propagation constant of the current on a beverage antenna comprising a bare, horizontal wire 12 inches above medium dry earth and terminated in a load impedance have been reported at a frequency of 18 MHz [23]. The measurements are in excellent agreement with an analytical model of King [24] and in poorer agreement with numerical results from the NEC-3 program. Burke has recently reported that NEC-2 (and NEC-3) predictions of the propagation constant are in good agreement with a theoretical model by Olsen, Kuester, and Chang [30].

3.2.5 Input Impedance, Directivity Pattern, and Absolute Gain of a Monopole Element with a Buried Radial-Wire Ground Plane

Measurements by Harnish, Lee, and Hagn of the input impedance of a monopole element with a buried radial-wire ground plane are in reasonable agreement with NEC-3 predictions [31].

3.2.6 Overall Assessment

The NEC-3 Sommerfeld integral option has inaccuracies as much as 25 percent or more in computing input current, input impedance, and radiation efficiency for antenna elements in close proximity to lossy earth (see sub-sections 3.2.2 and 3.2.3). However, the NEC-3 program in the Sommerfeld option is probably the best available model for antenna elements in close proximity to Earth with no ground screen.

SECTION 4

VERSION NEC-GS

4.1 CODE EXTENSIONS

4.1.1 Incorporation of Program SOMNTX

The double precision version of SOMNTX (D) was incorporated into the NEC-GS code for operation on a DEC VAX computer with 64-bit floating pointing double precision so that a separate input file is not required for each frequency and Earth complex permittivity.

4.1.2 Increase of Maximum Matrix Size

The maximum matrix size, as implemented by LLNL, limits the number of unknown current variables (or wire segments) to 50 for the NEC-GS version. MITRE extended the maximum matrix size by a factor of ten (to 500) for the NEC-GS version.

4.2 CODE CORRECTIONS

Version NEC-GS, prior to 1987, contained an error in the function ZINT in the subroutine LOAD of the card (LD) Loading. The subroutine LOAD is correctly given in the MFC-2 manual [25] but became corrupted in an early version of NEC-GS. This error has since been corrected by LLNL and by MITRE in its copy of the NEC-GS early version. The function ZINT computes the surface impedance of a round wire of free-space permeability when the conductivity of the wire is finite and is expressed in mhos/m. The input parameters to card (LD) are described in reference 25.

4.3 INPUT-OUTPUT FORMAT

4.3.1 Applicability of Antenna Geometry

Version NEC-GS is a more efficient version of NEC-3 for wire antennas that have rotational symmetry in the azimuthal direction. Examples of antennas with such a geometry are a vertical, electrically thick, dipole element; a vertical, electrically thick, monopole

element; and a monopole element whose ground plane consists of N uniformly-spaced radial wires, all of which may be in proximity to earth.

NEC-GS is a more efficient version for such a geometry because the input parameter specification is simplified and the matrix size (total number of wire segments or current variables) is reduced. For example, instead of specifying the coordinates for each segment of N radial wires, it is only necessary to specify the segment coordinate for a single wire. Furthermore, the matrix size for N radial wires with k segments/wire is reduced from kN to k when the number of rotations M equals N . The reduced matrix size, when combined with the extension of the maximum matrix size from 50 to 500 segments (see subsection 4.1.2), enables NEC-GS to model antennas with larger wires and a greater number of wires than can be modeled by NEC-3 with a matrix size of 1500 segments.

4.3.2 Input Parameter Specification

Input parameter guidelines are given in reference 7. The following guidelines [26] may also be of interest to the user.

Wire intersections are assumed to be connected if two wires are within each other by an amount of $1/1000$ of a segment length.

Horizontal wires on the air side of the earth interface should not approach the earth's surface to within the greater of $10^{-6}\lambda$ or 2 to 3 times the wire radius.

A monopole segment that is connected to a horizontal wire should be at least as short as the height of the horizontal wire above the earth's surface.

The physical junction of several radial wires with a vertical element is modeled as a singular point (a node) without regard as to whether the radial wires are conically tapered so that they are physically able to fit around the vertical element.

The wire currents at a mode are constrained to satisfy Kirchhoff's current law without regard for current leakage into the earth.

The format for the field of the input parameters, as illustrated on page 5 of reference 7, should be meticulously followed. For example, in the GR card that specifies the integer number of ground radials, the omission of the concluding comma increases the number of radials by a factor of ten.

In the NEC-3 and NEC-GS programs, the segment length should be at least four times longer than the segment radius. If not, the extended kernal option (IK card) should be used for segment lengths as small as one segment radius.

The difference in radii of two adjoining wire segments (or two wires at a junction) should be minimized. A method for minimizing the difference in radii is the tapering of segment radii along one of the adjoining wires.

A rotational model may be used to represent a vertical element of radius b by a cage of M vertical elements each of radius b_w along a circumference of radius b . Best results are obtained by $b_w = b/M$ so that the vertical elements have the same total surface area as the original element [27, 28]. Rotational model representations of a vertical dipole element, a vertical monopole element, and a monopole element with a radial wire ground plane -- all in proximity to earth -- are shown in figures 4 through 6, respectively. In figure 6, the number of rotations M is equal to the number of radial wires, and the radius of the rotational vertical elements is equal to the radius b_w of the radial wires.

4.3.3 Interpretation of Output Parameters

When the rotational model is not used ($M=1$), the output parameters represent those of the physical antenna. However, when the rotational model is used, the output parameters are those of the rotational elements and not those of the physical antenna. The algebraic operations required on the rotational model output parameters to obtain the output parameters for the physical antenna are summarized in table 8.

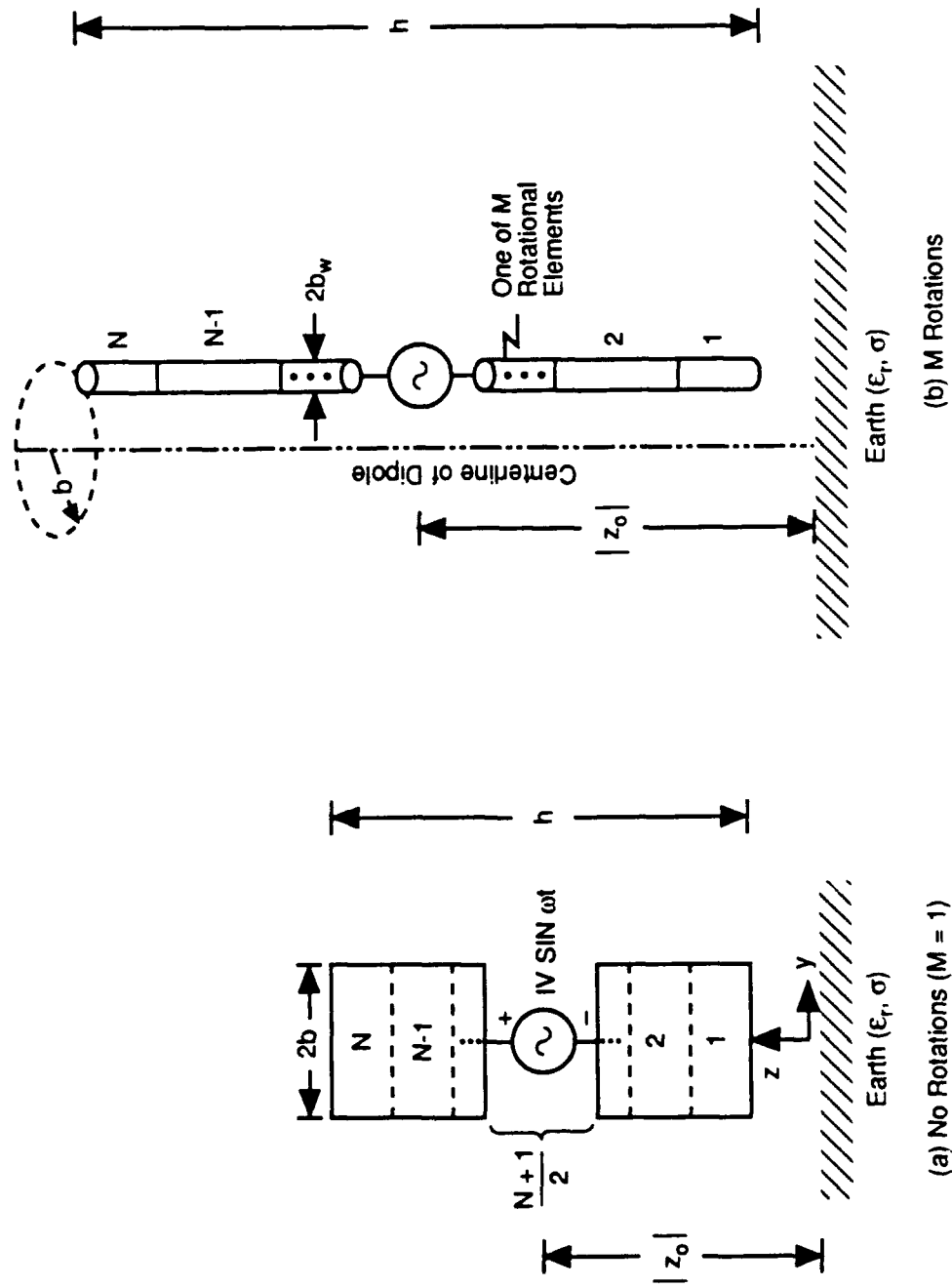
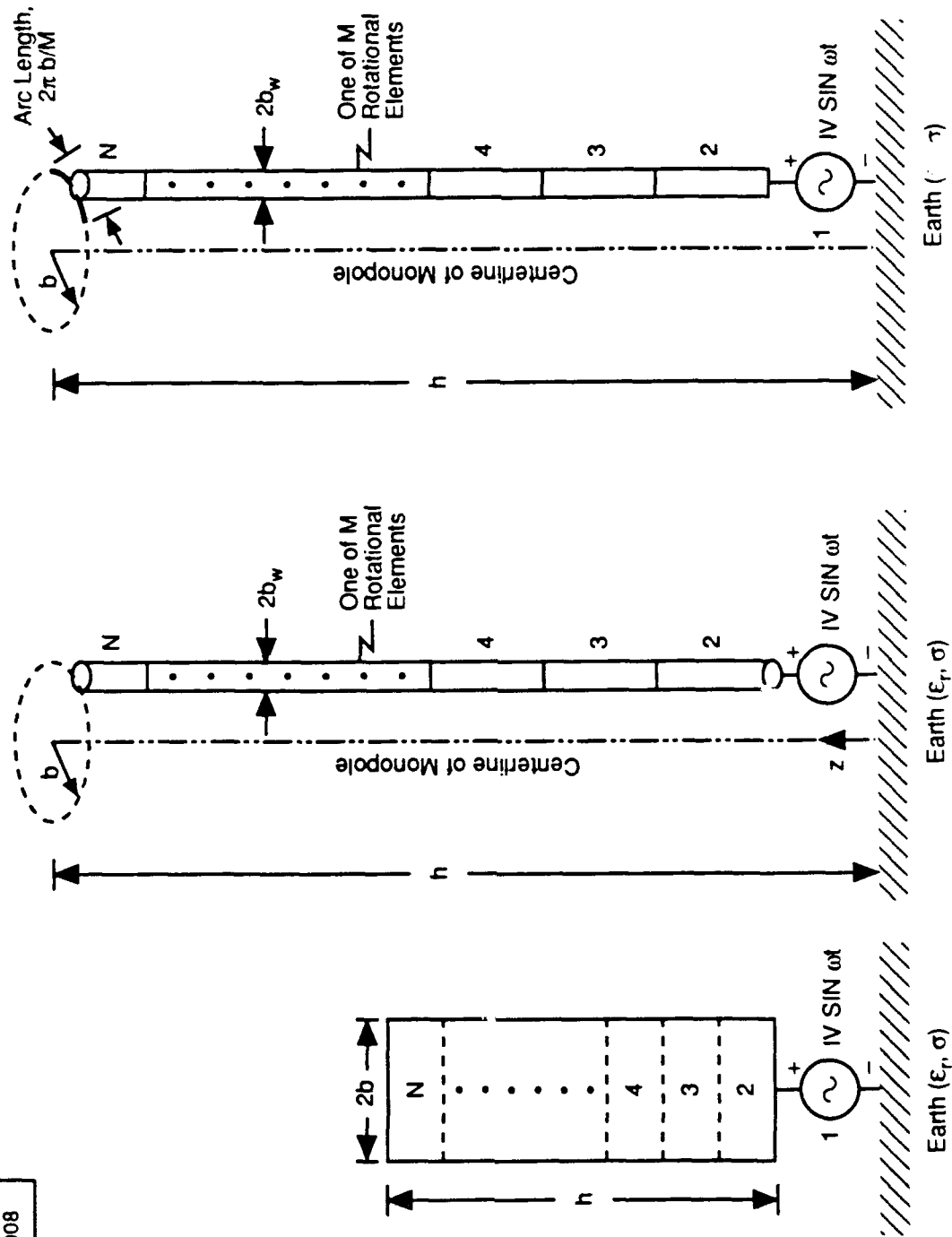


Figure 4. Rotational Model Representation of a Vertical Dipole Element in Proximity to Earth



(a) No Rotations ($M = 1$)

(b) M Rotations

(c) M Rotations with Connecting Link

Figure 5. Rotational Model Representation of a Vertical Monopole Element in Proximity to Earth

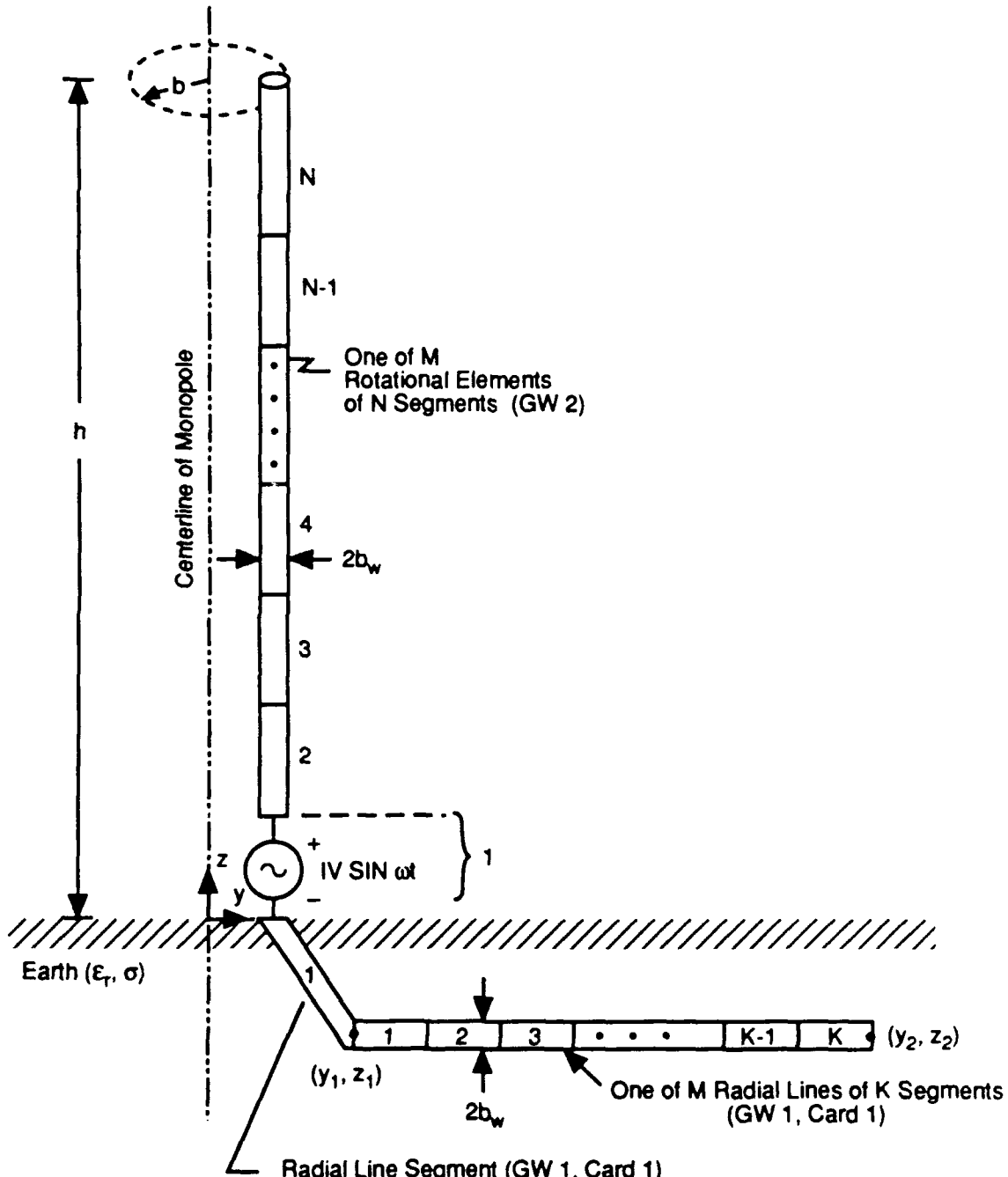


Figure 6. Rotational Model Representation of a Monopole Element of Radius b with a Ground Plane of M Radial Wires of Radius b_w in Proximity to Earth

FY0015

Table 8. Algebraic Operations to Obtain Output Parameters of Physical Antenna When Using a NEC-GS Rotational Model with M Rotations

Output Parameter of Physical Antenna	Operation Required on the Rotational Model Output Parameter
Current on vertical element (amperes)	Multiply by M
Current on radial wire (amperes)	As printed out
Input impedance of vertical element (ohms)	Divide by M
Input admittance of vertical element (mhos)	Multiply by M
Radiation efficiency* (numeric)	Divide by M
Gain (dB)	Subtract $10 \log_{10} M$

* Radiation efficiency = one-half of printed out value of average power gain for cases when the antenna is in proximity to lossy earth

4.4 COMPARISON WITH OTHER MODELS

4.4.1 LLNL Validation Efforts

LLNL has compared NEC-GS numerical results with theoretical results based on the compensation theorem by J. R. Wait and W. A. Pope for the input impedance of a quarter-wave monopole on a buried, radial-wire ground plane [10]. Good agreement was obtained between the two models only for those cases where implementation of the compensation theorem is expected to be valid, namely, for groundscreens of sufficient density (the number N of radial wires is large) and of sufficient extent (the length a of the radial wires are at least a wavelength in Earth). Unlike the NEC-GS method-of-moments model, the present implementation of the compensation theorem never solves the current on the ground plane, but instead assumes that the current distribution is the same as for a perfect ground plane. The inadequacy of the present implementation of the compensation theorem to yield accurate results of input impedance for small ground planes in proximity to earth was also pointed out by J. H. Richmond [19] when comparing his method-of-moments results for disk ground planes with results based on the compensation theorem by Wait and Surtees [29].

The NEC-GS method-of-moments program is probably the best available model for monopole elements with radial-wire ground planes (just as Richmond's method-of-moments program is the best available model for monopole elements with disk ground planes), provided that the ground planes are not so large that the maximum matrix size of the program is exceeded or that the computer run time is too excessive.

4.4.2 Comparison with NEC-3

This subsection compares numerical results obtained from NEC-GS with those obtained from NEC-3.

The test case, in the NEC-GS user's guide [7], is for a monopole element with six buried radial ground wires that have same radius as that of the monopole element. Test case numerical results, obtained from NEC-GS with no rotations ($M=1$), agree to within 0.01 percent of those obtained from NEC-3 (see table 9).

IY0016

**Table 9. Comparison of Numerical Results Obtained from
NEC-GS (M = 1) with Those Obtained from NEC-3**

Test case in G. J. Burke, "User's Guide Supplement for NEC-GS," Lawrence Livermore National Laboratory, Report UCRL-MA-107572, June, 1991.

Output parameter	Numerical value	
	NEC-GS	NEC-3
Element input current (amperes)	1.4279 E-2 - j7.7917 E-3	1.4278 E-2 - j7.7928 E-3
Radial wire input current (amperes)	2.279 E-3 - j1.375 E-3	2.2824 E-3 - j1.3754 E-3
Element input impedance (ohms)	5.3964 E+1 + j2.9446 E+1	5.39628 E+1 + j2.94524 E+1
Radiation efficiency (numeric)	0.291	0.291
Peak power gain	-0.27 dB	-0.27 dB
Direction of peak power gain	65 deg	65 deg

NEC-GS numerical results are compared with NEC-3 results in tables 10 through 13 for a dipole element, for a monopole element, and for thin and thick monopole elements with six buried radial ground wires, respectively. The corresponding NEC-GS rotational model geometries for these antennas are given in figures 4, 5, and 6, respectively. The numerical results for NEC-GS with no rotations ($M=1$) are almost identical in all cases to NEC-3 numerical results as is to be expected, since the model geometries are identical. Numerical results for NEC-GS rotational models ($M \geq 2$) have mixed agreement with NEC-3 numerical results.

The NEC-GS rotational model for the dipole element yields numerical results that are in almost exact agreement with those from NEC-3 when $M = 100$, corresponding to the case when the total surface area of the dipole rotational elements is equal to that of the physical dipole (see table 10).

For $M = 4$, the element input current differs by 25 percent from that from NEC-3 although other output parameters are in close agreement with NEC-3.

The NEC-GS rotational model for the monopole element yields numerical results that are within 3 percent of those for NEC-3 when $M = 100$, corresponding to the case when the total surface area of the monopole rotational elements is equal to that of the physical monopole (see table 11). However, at least as good agreement with NEC-3 is obtained when $M = 1000$. A connecting ring at the top of the rotation elements was added to see if the rotation element current would be modified. The addition of a connecting ring yields approximately the same results as without the connecting ring for a given value of M . This result is expected since the rotational symmetry constrains each rotational element to be at the same potential with no current flow in the connecting ring.

IY0017

**Table 10. Comparison of Dipole Element Numerical Results
Obtained from NEC-GS Rotational Models ($M = 1, 4, 8, 12, 16, 100$) with
Those Obtained from NEC-3.**

$h/\lambda = 0.250$, $b/\lambda = 1.667 \times 10^{-4}$, $b_w/\lambda = 1.667 \times 10^{-6}$, $l_{z0}/\lambda = 0.130$, $N = 21$ segments
 $\epsilon_r = 10.0$, $\sigma = 0.01$ S/m, $f = 5$ MHz ($\lambda = 60$ m)

Model		Element Input Current (Amperes after Multiplying by M)	Element Input Impedance (Ohms after Dividing by M)	Radiation Efficiency (Numeric after Dividing by M)	Power Gain (after Sub- tracting $10 \log_{10} M$) Peak/Direc- tion (dB/deg)
NEC-3		$0.712 \text{ E-4} + j0.155 \text{ E-2}$	$0.296 \text{ E+2} - j0.644 \text{ E+3}$	0.306	0.12 dB 67 deg
NEC- GS	$M = 1$	$0.715 \text{ E-4} + j0.155 \text{ E-2}$	$0.296 \text{ E+2} - j0.642 \text{ E+3}$	0.306	0.12 dB 67 deg
	$M = 4$	$0.532 \text{ E-4} + j0.134 \text{ E-2}$	$0.294 \text{ E+2} - j0.742 \text{ E+3}$	0.307	0.13 dB 68 deg
	$M = 8$	$0.632 \text{ E-4} + j0.146 \text{ E-2}$	$0.294 \text{ E+2} - j0.682 \text{ E+3}$	0.306	0.12 dB 67 deg
	$M = 12$	$0.665 \text{ E-4} + j0.150 \text{ E-2}$	$0.295 \text{ E+2} - j0.665 \text{ E+3}$	0.306	0.12 dB 67 deg
	$M = 16$	$0.681 \text{ E-4} + j0.152 \text{ E-2}$	$0.295 \text{ E+2} - j0.657 \text{ E+3}$	0.306	0.12 dB 67 deg
	$M = 100$	$0.712 \text{ E-4} + j0.155 \text{ E-2}$	$0.295 \text{ E+2} - j0.643 \text{ E+3}$	0.306	0.12 dB 67 deg

IY0018

Table 11. Comparison of Monopole Element Numerical Results Obtained from NEC-GS Rotational Models (M = 1, 4, 8, 12, 16, 40, 80, 100, 1000) with Those Obtained from NEC-3.

$h/\lambda = 0.250$, $b/\lambda = 1.667 \times 10^{-4}$, $b_w/\lambda = 1.667 \times 10^{-6}$, $N = 21$ segments
 $\epsilon_r = 10.0$, $\sigma = 0.01$ S/m, $f = 5$ MHz ($\lambda = 60$ m)

Model		Element Input Current (Amperes after Multiplying by M)	Element Input Impedance (Ohms after Dividing by M)	Radiation Efficiency (Numeric after Dividing by M)	Power Gain (after Sub- tracting $10 \log_{10} M$) Peak/Direc- tion (dB/deg)
NEC-3		0.354 E-5 + j0.157 E-3	0.144 E+3 - j0.638 E+4	0.151	-3.12/66
NEC- GS	M = 1	0.354 E-5 + j0.157 E-3	0.144 E+3 - j0.638 E+4	0.151	-3.11/66
	M = 4	0.215 E-5 + j0.122 E-3	0.145 E+3 - j0.822 E+4	0.153	-3.05/66
	M = 8	0.281 E-5 + j0.139 E-3	0.144 E+3 - j0.718 E+4	0.152	-3.07/66
	M = 12	0.305 E-5 + j0.145 E-3	0.144 E+3 - j0.688 E+4	0.152	-3.07/66
	M = 16	0.317 E-5 + j0.148 E-3	0.144 E+3 - j0.674 E+4	0.152	-3.07/66
	M = 40	0.336 E-5 + j0.153 E-3	0.144 E+3 - j0.655 E+4	0.152	-3.08/66
	M = 80	0.340 E-5 + j0.153 E-3	0.144 E+3 - j0.650 E+4	0.152	-3.08/66
	M = 100	0.341 E-5 + j0.154 E-3	0.144 E+3 - j0.650 E+4	0.152	-3.08/66
	M = 1,000	0.342 E-5 + j0.154 E-3	0.144 E+3 - j0.649 E+4	0.152	-3.05/66
	M = 4 with ring	0.215 E-5 + j0.122 E-3	0.146 E+3 - j0.822 E+4	0.153	-3.07/66
	M = 8 with ring	0.281 E-5 + j0.139 E-3	0.145 E+3 - j0.718 E+4	0.152	-3.07/66
	M = 12 with ring	0.304 E-5 + j0.145 E-3	0.144 E+3 - j0.688 E+4	0.152	-3.07/66
	M = 16 with ring	0.316 E-5 + j0.148 E-3	0.144 E+3 - j0.674 E+4	0.152	-3.07/66

IY0019

Table 12. Comparison of Numerical Results Obtained from NEC-GS Rotational Models (M = 1, 6) of a Thin Monopole Element with Radial-Wire Ground Plane with Those Obtained from NEC-3.

$h/\lambda = 0.250$, $b/\lambda = 1.667 \times 10^{-4}$, $N = 10$ segments (GW 2)
 $b_w/\lambda = 1.667 \times 10^{-6}$, $K = 14$ segments (GW1, Card 2), $(y_1, z_1) = (0.8 \text{ m}, -0.05 \text{ m})$
 $(y_2, z_2) = (12.0 \text{ m}, -0.05 \text{ m})$, $\epsilon_r = 10.0$, $\sigma = 0.01 \text{ S/m}$, $f = 5 \text{ MHz}$ ($\lambda = 60 \text{ m}$)

Output Parameter	Numerical Value		
	NEC-3	NEC-GS (M = 1)	NEC-GS (M = 6)
Element input current (amperes, after multiplying by M)	0.129 E-1 - j0.704 E-2	0.129 E-1 - j0.704 E-2	0.106 E-1 - j0.680 E-2
Radial wire input current (amperes, as printed out)	0.195 E-2 - j0.131 E-2	0.195 E-2 - j0.131 E-2	0.158 E-2 - j0.124 E-2
Element input impedance (ohms, after dividing by M)	5.980 E+1 + j3.272 E+1	5.980 E+1 + j3.272 E+1	6.695 E+1 + j4.307 E+1
Radiation efficiency (numeric, after dividing by M)	0.263	0.263	0.233
Peak power gain (dB, after subtracting $10 \log_{10} M$)	-0.72	-0.71	-1.24
Direction of peak gain (degrees)	65	65	65

IY0020

Table 13. Comparison of Numerical Results Obtained from NEC-GS Rotational Models (M = 1, 6) of a Thick Monopole Element with Radial-wire Ground Plane with Those Obtained from NEC-3.

$h/\lambda = 0.250$, $b/\lambda = 1.667 \times 10^{-3}$, $N = 10$ segments (GW2)
 $b_w/\lambda = 1.667 \times 10^{-5}$, $K = 14$ segments (GW1, card 2), $(y_1, z_1) = (8.0 \text{ m}, -0.05 \text{ m})$
 $(y_2, z_2) = (12.0 \text{ m}, -0.05 \text{ m})$, $\epsilon_r = 10.0$, $\sigma = 0.01 \text{ S/m}$, $f = 5 \text{ MHz}$, $(\lambda = 60 \text{ m})$

Output Parameter	Numerical Value		
	NEC-3	NEC-GS (M = 1)	NEC-GS (M = 6)
Element input current (amperes, after multiplying by M)	0.130 E-1 - j0.686 E-2	0.130 E-1 - j0.686 E-2	0.122 E-1 - j0.689 E-2
Radial wire input current (amperes, as printed out)	0.202 E-1 - j0.125 E-2	0.202 E-1 - j0.125 E-2	0.187 E-1 - j0.125 E-2
Element input impedance (ohms, after dividing by M)	6.015 E+1 + j3.171 E+1	6.016 E+1 + j3.171 E+1	6.223 E+1 + j3.523 E+1
Radiation efficiency (numeric, after dividing by M)	0.279	0.279	0.263
Peak power gain (dB, after subtracting 10 log ₁₀ M)	-0.44	-0.44	-0.71
Direction of peak gain (degrees)	65	65	65

The NEC-GS rotational model with $M = 6$ for a thin monopole element with six buried radial ground wires yields numerical results that differ from those for NEC-3 by eighteen percent for the monopole input current, by nineteen percent for the radial-wire input current, by twelve percent for the input impedance, by eleven percent for the radiation efficiency, and by 0.5 dB for the peak power gain (see table 12). The total surface area of the monopole rotational elements is six percent ($Mb_w / b = 6 \times 1.667 \times 10^{-5} / 1.667 \times 10^{-3} = 0.06$) of that of the physical monopole.

The NEC-GS rotational model with $M = 6$, for a monopole element and radial wires whose diameters are ten times larger than those in table 12, yields numerical results that differ from NEC-3 by six percent for the monopole input current, by seven percent for the radial-wire input current, by two percent for the input impedance, by six percent for the radiation efficiency, and 0.3 dB for the peak power gain (see table 13). It is not clear why close agreement with NEC-3 is obtained for the thick monopole element whose rotational elements have the same total surface area relative to the physical element (six percent) as for the thinner element in the preceding table.

4.4.3 Richmond's Method-of-Moments

NEC-GS results (for a 128 radial-wire ground plane) are in close agreement with results from Richmond's method-of-moments program RICHMOND4 [17, 19] (for a disk ground plane) when computing the radiation efficiency of a quarter-wave monopole element with a small ground plane on or in close proximity to medium dry Earth (see figure 7).

4.4.4 Overall Assessment

Version NEC-GS is a more efficient version of the NEC-3 Sommerfeld integral option for wire antennas that have rotational symmetry in the azimuthal direction (see subsection 4.3.1). The NEC-GS rotational model gives close agreement with NEC-3 when the total surface area of the rotational element is equal to the surface area of the physical element (see subsection 4.4.2). When this condition is not satisfied, inaccuracies of ten percent or more can occur in input current, input impedance, and radiation efficiency (see tables 12 and 13).

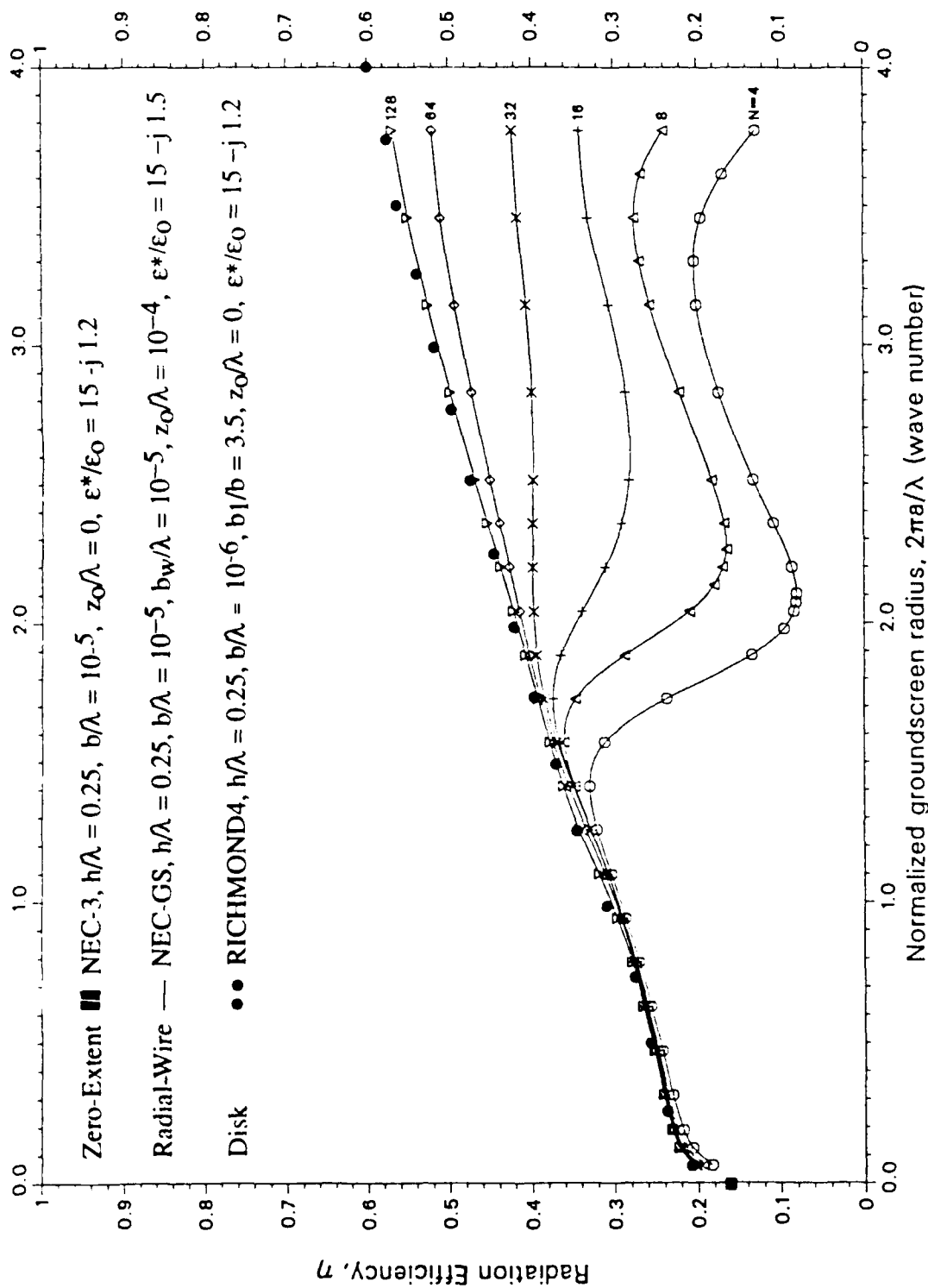


Figure 7. Radiation Efficiency of a Quarter-Wave Monopole Element with Different (Zero-Extent, Radial-Wire, and Disk) Ground Planes on or just above Medium Dry Earth

The format for the field of the input parameters is somewhat user unfriendly because the omission of a concluding comma in the GR card increases the number of radials by a factor of ten (see subsection 4.3.2). NEC-3 results of radiation efficiency, for ground planes with a large number of radial wires, agree with results from Richmond's method-of-moments for a disk ground plane.

SECTION 5

VERSION NEC-3I

5.1 CODE EXTENSIONS

5.1.1 Incorporation of Program SOMNTX

The single precision version of SOMNTX was incorporated into the NEC-3I code for operation on a FPS-M64 computer with 64-bit floating point single precision so that NEC-3I no longer requires a require a separate input file for each frequency and Earth complex permittivity.

5.1.2 Increase of Maximum Matrix Size

The maximum matrix size, as implemented by LLNL, limits the number of unknown current variables (or wire segments) to 300 for the NEC-3I version. MITRE extended the maximum matrix size by a factor of five (to 1500) for the NEC-3I version.

5.3 LLNL VALIDATION EFFORTS

LLNL has compared NEC-3I numerical results with theoretical results by R. W. P. King, T. T. Wu, and L. C. Shen for the complex wave number of a horizontal, insulated wire above conducting earth [10]. Excellent agreement was obtained between the two models. The model by King, et al., had previously been shown to give good agreement with measurements by R. M. Sarbello, et al., for a wire over water.

LLNL has also compared NEC-3I numerical results with theoretical results by J. R. Wait for the propagation constant of the current on a horizontal, insulated wire buried in conducting earth [10]. The two models agree to within 0.2 percent for the phase constant over the large range of burial depths, but differ by as much as 1 percent for the attenuation constant at small burial depths.

SECTION 6

CONCLUSIONS

The NEC-3, NEC-GS, and NEC-3I versions of the Numerical Electromagnetics Code are method-of-moments computer programs developed by Lawrence Livermore National Laboratory for predicting the performance of wire-element antennas above or buried in flat earth.

The Fresnel reflection coefficient option of NEC-3 yields poor results for input current, input impedance, and radiation efficiency. Correct results for directive gain are obtained for the case of an element (with no ground plane) in proximity to earth.

The NEC-3 Sommerfeld integral option, with its NEC-GS version, is probably the best available model for monopole elements with radial-wire ground planes (just as Richmond's method-of-moments program is the best available model for monopole elements with disk ground planes) provided that the ground planes are not so large that the maximum matrix size of the program is exceeded or that the computer run time is too excessive. Inaccuracies as much as 25 percent or more occur in computing input current, input impedance, and radiation efficiency for antenna elements in close proximity to lossy earth.

Version NEC-GS is a more efficient version of the NEC-3 Sommerfeld integral option for wire antennas that have rotational symmetry in the azimuthal direction. The NEC-GS rotational model gives close agreement with NEC-3 when the total surface area of the rotational elements is equal to the surface area of the physical element. When this condition is not satisfied, inaccuracies of ten percent or more can occur in input current, input impedance, and radiation efficiency. The format for the field of the input parameters is somewhat user-unfriendly because the omission of a concluding comma in the GR card increases the number of radials by a factor of ten.

Version NEC-3I extends NEC-3 to include the case of insulated wires. NEC-3I numerical results are in excellent agreement with analytical models for the propagation constant of a horizontal insulated wire above and below conducting earth.

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